Int. J. Heat Mass Transfer Vol. 34, No. 1, pp. 320–322, 1991
Printed in Great Britain

0017-9310/91 $3.00+0.00
Pergamon Press plc

Thermophoretic deposition near the leading edge of cylindrical surfaces

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(*Received April* 1984 *and in final form* 14 *February* 1990)

INTRODUCTION

The deposition of sub-micrometre solid particles on the leading edge region of cylindrical surfaces is a serious problem for a number of applications. Of current concern are the turbine blade leading edge in gas turbines, the superheater tubes in boilers and the embedded tubes in fluidized beds. There exist a number of mechanisms for particle deposition (e.g. Brownian diffusion, thermophoresis, convection, inertial impact, sedimentation, etc.), but for particles in the region $0.1 < d_p < 3$ μm (which is the expected range for most of the particles to pass through a turbine after filtering the combustion flue gases) thermophoresis has been proved, in general, to be the most significant [1]. Experimental data indicate that the flue gases must be diluted, with particle concentrations of the order of 10 p.p.m. [2]. Another significant implication of the thermophoretic diffusion is that small particles are deflected from the flowstream lines, so that the selection of the proper diameter in laser Doppler velocimetry, for such a flow field, has to be made for particles below the diameter which leads to inertial impact on the cylinder and above the one that is subject to a strong thermophoretic attraction.

The behaviour of particles with $d_p \geqslant 5$ μm, over cylinders in isothermal flow conditions has been studied by Laitone [3] and in non-isothermal flow conditions by Kladas and Georgiou [4] for $d_p \geqslant 3$ μm.

The present note investigates the trajectories and impact conditions of sub-micrometre solid particles ($0.05 \leqslant d_p \leqslant 1$ μm) in the region near the leading edge ($0 \leqslant \phi \leqslant 40°$) of cylinders representing turbine blade leading edges and superheater tubes in their respective, typical, flow and thermal environments. The trajectories are estimated by considering inertial, drag and thermophoretic forces only.

ANALYSIS

The flow considered is that around a cylinder, as shown in Fig. 1, split into an inviscid and a viscous region. The inviscid solution is well known and given in many textbooks, e.g. ref. [5]. The viscous boundary layer was calculated by incorporating the STAN-5 code [6], modified by the inclusion of a curvature effect on the mixing length as suggested by Adams and Johnson [7]. The code was calculated for $\phi \geqslant 2°$. For $0 \leqslant \phi \leqslant 2°$ a Falkner–Skan solution was given. The velocity distribution in the free-stream of the boundary layer was that of Hiemenz [8]. The region of interest was $0 \leqslant \phi \leqslant 40°$. In the typical Reynolds number range based on the turbine blade leading edge diameter and the superheater tube diameter ($4 \leqslant \log_{10} Re_{d_p} \leqslant 5$), the data presented by Schlichting [9] indicate that the boundary layer is laminar. The code considers the fluid viscosity, conductivity and specific heat to be functions of temperature. The particle trajectories were calculated by employing the equation:

$$\frac{1}{6}\pi\rho_p d_p^2 \frac{d\mathbf{U}_p}{dt} = \mathbf{F}_D + \mathbf{F}_{TH} \tag{1}$$

in its two components along the directions (x, y). The particle temperature was calculated from the equation

$$\frac{1}{6}\pi\rho_p d_p^3 C_p \frac{dT_p}{dt} = \pi d_p^2 h(T - T_p). \tag{2}$$

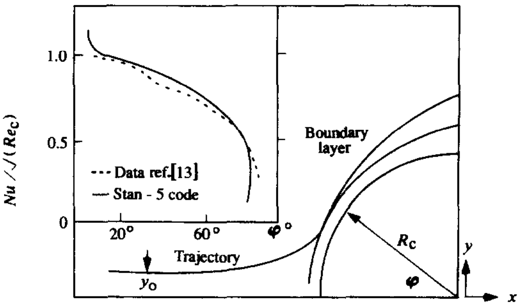

FIG. 1. The flow field and the validation of the computational code.

In equations (1) and (2), $\mathbf{F}_D$ and $\mathbf{F}_{TH}$ are the drag and thermophoretic forces, and h the heat transfer coefficient. The drag force was calculated from the particle drag law of Stokes, as shown in ref. [10]. The thermophoretic drag was calculated by employing the Brock equation [11], which was shown in the experiments of Talbot *et al.* [12], to give the best results in laminar boundary layers. The heat transfer coefficient was taken from the following Nusselt number relationship:

$$Nu_{d_p} = 2 + 0.236 Re_{d_p}^{0.6} Pr^{0.33}. \tag{3}$$

The particle was assumed to be in equilibrium with flow at a distance of $x = -2R_c$.

The equations were integrated by using a fourth-order Runge–Kutta scheme, with a very small time step.

RESULTS

Two types of flow conditions were considered in the calculations. The first was that representing the leading edge of the high pressure turbine and the second the superheater tubes in a steam generator. The values of the respective parameters are given in Table 1. The particle diameters were in the region $0.05 \leqslant d_p \leqslant 1$ μm and are presented in the figures in non-dimensional form, as Stokes numbers (StK), where

$$StK = \frac{\rho_p d_p^2 U_\infty}{18\mu R_c}. \tag{4}$$

The particles were assumed to be ash with the following characteristics: $C_p = 1.300$ J kg^{-1} K^{-1}, $\rho_p = 2500$ kg m^{-3}, $K_p = 0.25$ W m^{-1} K^{-1}. Impact was assumed when the calculated values of X_p and Y_p gave

$$\surd(X_p^2 + Y_p^2) \leqslant R_c + R_p \tag{5}$$

NOMENCLATURE

c	specific heat
d	diameter
$\mathbf{F}$	force
h	heat transfer coefficient
K	thermal conductivity
J	momentum
Nu	Nusselt number
Pr	Prandtl number
R	radius
Re	Reynolds number
StK	Stokes number
T	temperature
t	time
X	X-direction
Y	Y-direction.

Greek symbols

β	impact angle
δ	boundary layer thickness
μ	viscosity
ρ	density
ϕ	angle, shown in Fig. 1.

Subscripts

c	cylinder
D	drag
en	entrance to the boundary layer
i	impact
o	initial
p	particle
w	wall
∞	infinite.

Table 1. Flow parameters

	T_∞ (K)	T_w (K)	U_∞ (m s^{-1})	R_c (m)	P (bar)
(i) Gas turbine blade leading edge	1600	1100	200	0.01	15
(ii) Superheater tube	1273	973	30	0.05	1

and the particles were assumed to enter the boundary layer when

$$\sqrt{(X_p^2+Y_p^2)} \leqslant R_c+\delta. \tag{6}$$

The accuracy of the code was checked against the experimental data of Achenbach [13], for $Re_0 = 10^5$, and was shown to be very good for $\phi \geqslant 7^\circ$ (Fig. 1). The emphasis was given on the particles near the capture height, which gave impacts above this value of ϕ. The influence of the thermophoretic force on the trajectory of the particles, inside the boundary layers, is given in Fig. 2. As it can be seen, the capture height is very small, of the order of $(y_0/R_c) \simeq 0.001$. Within this height, however, all particles with $d_p \leqslant 1$ μm will impact on the surface. For the smaller particles, the thermophoretic attraction is very strong, resulting in a sharp deflection, compared to the direction of the streamlines (trajectories without the influence of $\mathbf{F}_{TH}$). Even the larger particles ($StK \simeq 0.02$) have significantly influenced paths, when they do not impact. This means that only particles in the

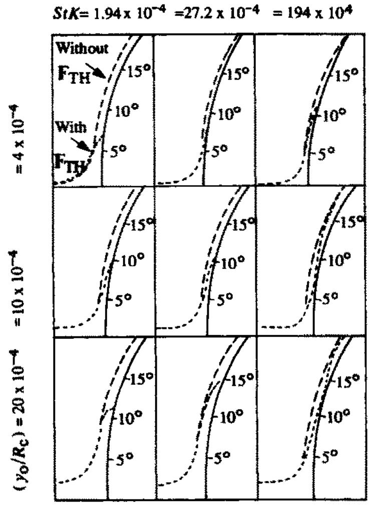

FIG. 2. The particle trajectories. The influence of the thermophoretic force.

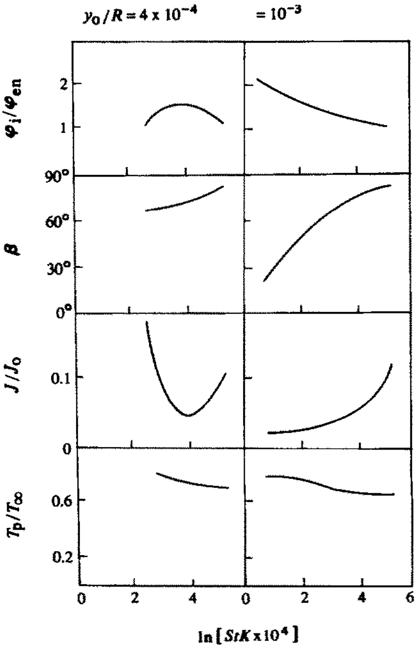

FIG. 3. The impact conditions.

range $2 \leqslant d_p \leqslant 3$ μm are reliable for LDV measurements in these conditions.

The conditions at impact are given in Fig. 3. The ratio (ϕ_i/ϕ_{en}) between the angle ϕ that the particles impact the cylinder and the corresponding one they enter the boundary layer, is of the order of 1, increasing towards 1.5 for the larger particles. The angle of impact β (with respect to the local tangent on the cylinder) is nearly 90° for the smaller particles and drops almost linearly to 30° for the larger ones. The ratio of the particle momentums between impact and the initial point, is of the order of 5–15%, the particles nearer to the axis exhibiting a larger reduction. The particle temperature at impact for the smaller particles is close to T_w (indicative of a very fast cooling) but for the larger particles is somewhat in the middle between T_w and T_∞.

CONCLUSIONS

The deposition of sub-micrometre ash particles near the leading edge of cylindrical surfaces, under conditions representative of gas turbine blades and boiler superheater tubes has been investigated by calculating their trajectories and impact conditions.

The results indicate:

(i) The influence of the thermophoretic force is drastic. Use of sub-micrometre particles in LDV measurements in such flow fields should be avoided.

(ii) Such particles upon entering the boundary layer are attracted very quickly towards the surface. Their momentum, however, has been reduced significantly at impact, although their temperature is somewhere in the middle between the inviscid flow temperature and the wall temperature. Thus, although they impact in a normal direction, they do not have enough momentum to rebound.

(iii) The capture height of these particles is very small, when compared to the cylinder radius, being of the order of 0.001.

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Int. J. Heat Mass Transfer Vol. 34, No. 1, pp. 322–327, 1991
Printed in Great Britain
0017-9310/91 $3.00+0.00

Laminar-flow heat transfer to a fluid flowing axially between cylinders with a uniform wall heat flux

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(*Received* 5 *November* 1989 *and in final form* 14 *February* 1990)

1. INTRODUCTION

IN REGARD to the laminar-flow heat transfer to a fluid flowing axially between cylinders, several analytical [1, 2] and numerical [3–5] asymptotic solutions applicable in the region of large axial distance are available for a triangular array [1–5] and a square array [5] of cylinders with a uniform wall temperature peripherally and a uniform wall heat flux axially [1, 2, 4], and with a uniform wall heat flux peripherally and axially [2–5]. However, little is known about the characteristics of axially varying heat transfer in this geometry, although such characteristics are required for the design of multitubular heat exchangers for highly viscous liquid and rod-bank regenerators.

Thus, in the previous paper [6], the characteristics of axially varying heat transfer to a fluid flowing axially between a triangular array or a square array of cylinders with a uniform peripheral and axial wall temperature were analyzed using a finite-difference technique. Here, attention is directed to analyzing the case of cylinders with a uniform peripheral and axial wall heat flux.